Kuwait University — Math. 101 — Jan. 12, 2008
Dept. of Math. & Comp. Sci. — Final Exam. — Duration: two hours

---

Answer the following questions. Each question carries 4 points.
Calculators, mobile phones and pagers are not allowed.

1. Find the value of the constant A so that the function

$$f(x) = \begin{cases} 2x^2 - \frac{x^2-16}{\sqrt{x}-2} & , \quad x > 4 \\ A + \cos(x-4) & , \quad x \le 4 \end{cases}$$

is continuous at $x = 4$.

2. Evaluate the following limit, if it exists:

$$\lim_{x\to 2} \left[\frac{6x^2 - 11x - 2}{x^3 - 8} + (x-2)^2 \sin \frac{1}{(x-2)^2} \right].$$

3. Use the chain rule to find $\frac{dy}{dt}$ at $t = \frac{\pi}{4}$, where

$$y = \sqrt[3]{u^2 + u - 1} \quad \text{and} \quad u = \sec^2 t - \frac{2}{\sqrt{3 + \tan t}}$$

4. Find the maximum area of the rectangle that can be inscribed inside a semi-circle of radius 2.

5. Evaluate the integrals:

(a) $\int_{-\pi}^{\pi} \left(x^3 + \sqrt{\pi^2 - x^2} \right) \, dx.$

(b) $\int \frac{(1 + \cot 7x)^9}{\sin^2 7x} \, dx.$

6. Show that $f(x)$ is increasing on $[1, \infty)$, where

$$f(x) = \int_1^{\sqrt{x}} \sqrt{1 + t^4} \, dt \; + \; \int_0^{\frac{\pi}{2}} x^3 \sin^2(x^2 + 1) \, dx.$$

7. State the mean value theorem for definite integrals and apply it to find a number $c \in (-2, -1)$ that satisfies the conclusion of this theorem for the integral $\int_{-2}^{-1} \frac{8}{x^3} \, dx.$

8. Find the average value of the function $f(x) = 3x|x - 1|$ on the interval $[0, 2]$.

9. Find the area of the region bounded by the curves $y = 3x^2$ and $y = 4 - x^2$.

10. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line $y = 2$.